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Serviceability of non-prismatic concrete beams:

combined-interaction method

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Abstract

Interest in the shape optimisation of concrete members is increasing alongside the availability of fabric formwork as a relatively simple technique to cast non-prismatic concrete structures. Research has shown that up 40% of concrete can be saved when shape optimised concrete beams are cast in fabric forms. However, optimisation results in members with non-uniform cross-sections and the resulting beam is less stiff than an equivalent strength prismatic beam. Serviceability, rather than strength, may govern the design of such members and therefore understanding the serviceability behaviour (deflection and cracking) of

shape optimised concrete members becomes is a critical design consideration. There are many methods which can be used to evaluate serviceability behaviour of reinforced concrete beams, including the full-interaction method, which assumes no slip between the reinforcement and the surrounding concrete, and the partial-interaction method which accounts for slip. The full-interaction method is based on a smeared crack approach and so is unsuited for the prediction of cracking behaviour. The partial-interaction method, on the other hand, assumes that cracks form through bond-stress transfer only. In the case of non-prismatic concrete beams, the cracking capacity varies along the member. Therefore, cracking can occur over extended regions (full and partial bond interaction regions) and so it can be argued that neither of these models is fully suitable for the prediction of deflections and cracking of shape-optimised concrete beams. In this paper, a novel combined-interaction method is, for the first time, presented to predict the serviceability behaviour of non-prismatic statically determinate concrete beams by simulating both full and partial bond interactions at different cracked and uncracked regions along the length of the member. In order to validate this approach, two non-prismatic simply supported beams were cast and tested. The test results for deflections, crack widths and crack spacings were in good agreement with the predicted results.

Keywords: Combined-interaction method, Fabric-formed concrete, full-interaction analysis, partial-interaction analysis, serviceability.

1 Introduction:

Fabric formwork, a technique for casting non-prismatic concrete members, has recently brought increased attention to the area of shape optimisation of concrete structures (Veenendal et al., 2011). Figures 1 & 2) show a number of fabric-formed concrete beams and trusses. Shape optimised concrete members use less concrete material than their prismatic counterparts because their shape reflects the requirements of the loading envelope (Bailiss, 2006, Orr, 2012). Such members, generally have smaller cross-sectional dimensions along their length than an equivalent strength prismatic beam (Garbett et al. 2008). However, this makes them significantly less stiff and, in many cases, serviceability criteria rather than strength can govern their design (Tayfur, 2017). Therefore, understanding the serviceability behaviour of non-prismatic fabric-formed concrete members is important.

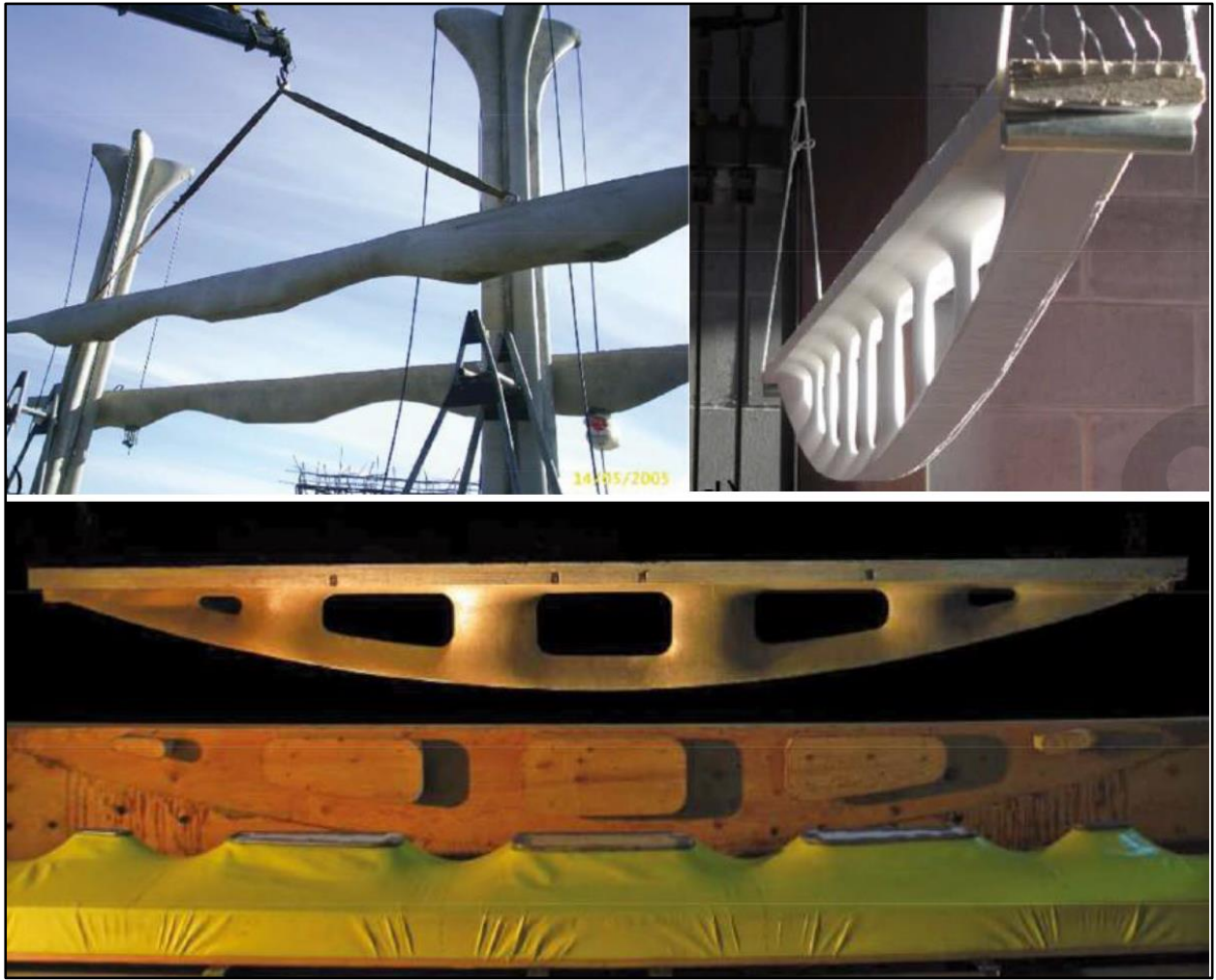


Figure 1: Fabric-formed concrete (Photos by Mark West: Hawkins et al. 2016)

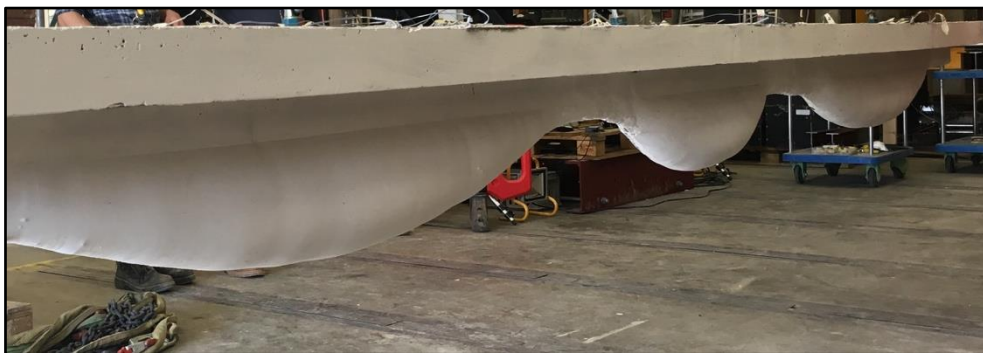


Figure 2: Fabric-formed continuous concrete beam cast at the University of Bath (credit Tayfur, 2016)

Common methods used to evaluate short-term serviceability behaviour of concrete beams include empirical relationships and sectional analysis. The empirical formula suggested by ACI committee 435 (2000) to find deflections combines cracked and uncracked flexural stiffness to find an effective flexural stiffness (EI_e), and applies this value to the whole beam. These methods are not suitable for optimised non-prismatic beams

as flexural stiffness varies significantly along the beam length due to changes in cross-section (and specifically section depth). Therefore, a sectional approach is seen to be more appropriate as it allows every section along the member to be dealt with separately.

Sectional analysis methods can be divided into two categories; full-interaction and partial-interaction. The full-interaction method (FI) is based on the assumption of perfect bond in which plane sections remain plane and no slip is assumed between the reinforcement and concrete (Kwak and Kim, 2002). The full-interaction method is unable to predict specific cracking behaviour of concrete beams because the cracks are assumed to be smeared along the cracked region of the member. The partial-interaction method (PI), on the other hand, allows slip to occur between the reinforcement and the concrete (Oehlers et al. 2011, Visintin et al.,2013).

Partial-interaction analysis has previously only been applied to prismatic beams (Visintin et al., 2013). The initial crack is formed first under the assumption of full-interaction, and subsequent cracks are assumed to form purely due to partial-interaction (Oehlers et al., 2013). For prismatic simply supported beams under uniformly distributed loading, an initial crack occurs at the point of maximum bending moment, and all other subsequent cracks occur due to concrete-reinforcement bond stress transfer when the tensile strength of the concrete is exceeded. The bending moment value decreases when moving away from the initial crack but the cracking capacity of the beam stays constant, assuming that the sectional properties do not change along the beam length. The primary cracks therefore can only occur through partial-interaction at a point at which the full-interaction region starts.

Crack development in non-prismatic beams is more complicated since the sectional cracking capacity of the beam is not constant. This, coupled with the variable applied moment, means that cracking can potentially be initiated at any location, and subsequent cracking can occur over extended areas. The formation of these cracks cannot be predicted purely by the partial-interaction method because they form due to a full-interaction mechanism. Conversely, the full-interaction method cannot simulate crack-widening, preventing its use for calculating crack spacing and crack widths. Thus, neither of the interaction models can be applied alone to accurately model the cracking behaviour, and a method that takes into account different interaction regions is required to find the order of crack formation and the location and widths of cracks. Therefore, in this paper, a novel combined-interaction model is developed to

simultaneously simulate full and partial interaction behaviour between the reinforcement and the surrounding concrete by which the behaviour of non-prismatic and statically determinate concrete members can be predicted.

2 Methodology

2.1 Full-interaction analysis

The full-interaction (FI) model assumes that the member deflects under flexural curvature only, and that cracks are smeared for the purpose of finding curvatures. Concrete cracks when its tensile strain is exceeded. Using this model, the moment-curvature relationship at every section of the member is found step by step, by dividing each section into a number of horizontal strips, Figure 3. Increments of curvature can be applied gradually while adjusting the neutral axis to ensure equilibrium in order to determine the moment of resistance.

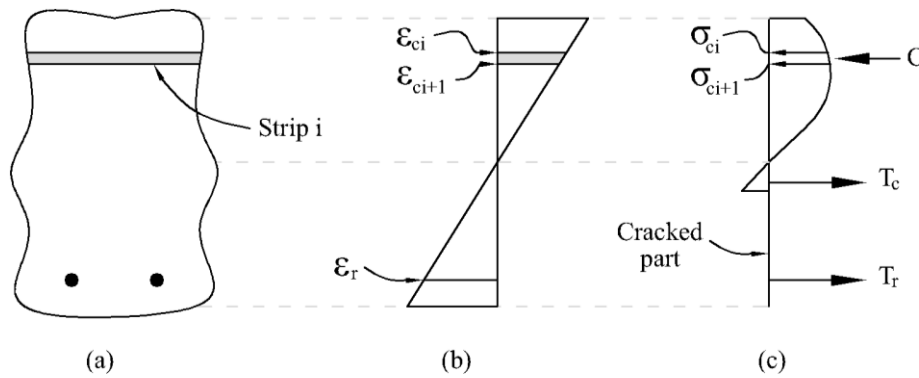


Figure 3: Full-interaction analysis (a) discretised section, (b) strain profile, (c) stress profile

2.2 Partial-interaction analysis

The partial-interaction model was previously developed by Visintin et al. (2013) to evaluate the moment-rotation relationship at any section using a numerical procedure that finds the relationship between the force in the reinforcement, P , and the slip Δ . This sectional-segmental analysis model assumes an initial crack already exists in the beam, the location of which can be found initially from full-interaction analysis. Following the initial crack, the partial-interaction method is able to track the formation and propagation of primary, secondary and tertiary cracks under increasing load (for prismatic beams). A primary crack is a

crack which forms due to reinforcement-concrete bond-stress transfer where the full-interaction boundary conditions are met, which is where the slip and slip-strain values are zero at the same point. A secondary crack occurs between two existing cracks where the slip value is zero but slip-strain has a non-zero value. At latter stages of loading, and if the reinforcement–concrete bond strength suffices, tertiary cracks may form between two secondary cracks.

The $P - \Delta$ relationship is found from partial-interaction analysis at a cracked section, and a sectional analysis procedure is then employed to find the moment-rotation relationship. As shown in Figure 4, the formation of a crack introduces a rigid-body rotation, 2θ , in the cracked region due to crack opening 2Δ (equal to the slip of the reinforcement), and an associated displacement in the compression part of the concrete equal to $2d_c$. The variation of deformation in the compression part of the concrete is assumed to be linear, and d_c is proportional to the value of slip at the crack and neutral axis depth (Oehlers et al., 2013).

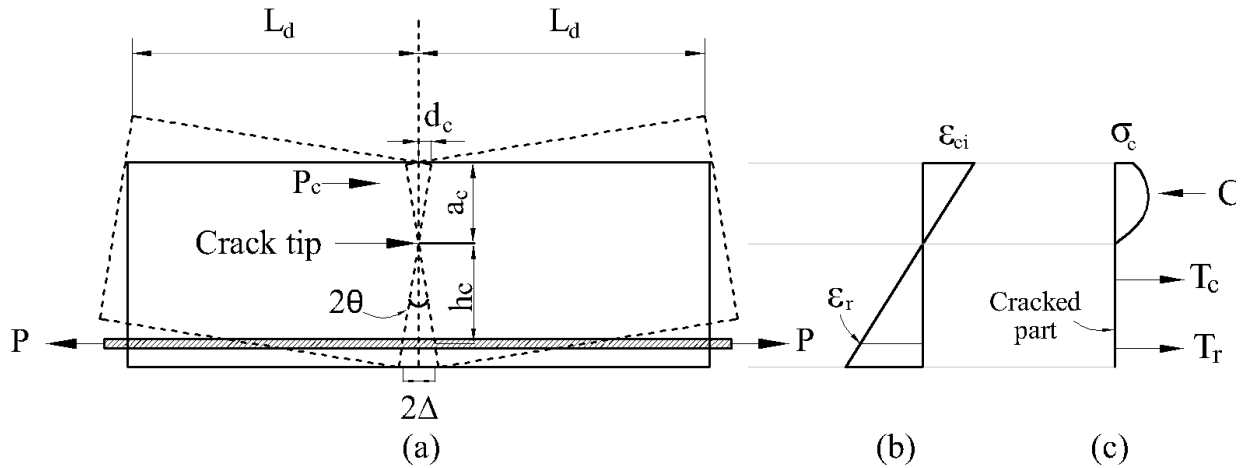


Figure 4: (a) Deformations at a crack, (b) Strain profile, (c) Stress distribution

Initially, the depth to the neutral axis a_c is estimated, so that the deformation in the concrete d_c and the moment corresponding to the rotation θ can be calculated from the transformed section as follows:

$$d_c = \frac{\Delta}{(d-a_c)} a_c \quad (1)$$

The deformation in the concrete d_c can be converted to a strain profile by dividing it by the deformation length L_d , which is taken as equal to half of the crack spacing (distance between two subsequent cracks):

$$\varepsilon_c = \frac{d_c}{L_d} \quad (2)$$

Once the strain profile is obtained, the same sectional analysis procedure defined previously for full-interaction analysis can be applied iteratively to find the correct value of the initially estimated a_c , and subsequently to determine the value of moment M induced by the slip Δ .

2.3 Combined-interaction method

2.3.1 Cracking

A new combined interaction method is developed in this paper to predict cracking and deflections for non-prismatic concrete beams. In the pre-cracking stage, full-interaction analysis is applicable to find deflections and predict the location of the initial cracks and the bending moment at which each crack forms. At later stages of cracking, a partial bond interaction at the vicinity of cracks (where slip is not zero) and full bond interaction elsewhere along the beam is seen. Therefore, subsequent cracks may either be identified through full- or partial-interaction methods.

Figure 5 illustrates the cracking process of a non-prismatic beam within the different interaction regions. Assuming that a is an initial crack discovered through full-interaction analysis, in Figure 5(a), then the slip distribution between the reinforcement and the concrete to the left and right of the crack is as shown in Figure 5(b). The next crack occurs when either 1) the applied bending moment at the initial crack results in a stress transferred to the concrete via bond-slip and partial-interaction mechanism to cause a crack at the primary crack spacing distance (S_p); or 2) the applied bending moment exceeds the cracking moment capacity (M_{Crf}) anywhere in the full-interaction region where slip is zero. In this stage, b and c are the two possible cracks on either side of the initial crack a that may form based on one of mechanisms given above.

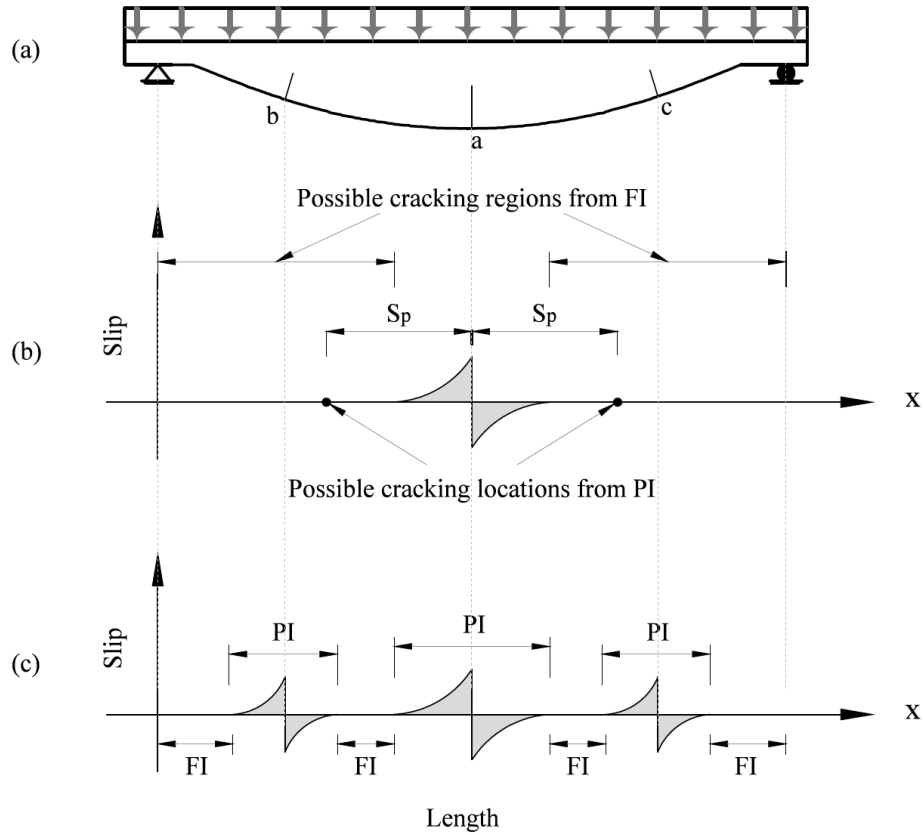


Figure 5: Cracking process in the proposed combined-interaction method: (a) Optimised concrete beam, (b) Initial crack, (c) Three cracks

Similarly, as shown in Figure 5(c), the next crack may occur somewhere in the remaining full-interaction regions, marked as FI, where the ratio of applied moment (M_A) to cracking moment capacity (M_{crf}), (M_A/M_{crf}), is largest, or in the partial-interaction regions, marked as PI, where the applied moment at one of the cracked sections exceeds the moment required to cause a crack through bond-stress transfer, (M_{Crf}).

The moment-curvature behaviour of the cracked sections depends mostly on their distance from neighbouring cracks (i.e. bond stress transfer length). Larger crack spacing results in higher total bond stress transfer and eventually larger slip values at the same load level when compared with closer crack spacing, provided that the crack spacing is smaller than S_p . Furthermore, the value of bending moment at which a section cracks due to partial-interaction depends on the distance over which the bond stress transfer occurs. Likewise, the locations of cracks formed in the full- or partial-interaction regions at various stages of loading dictate where the next generation of cracks will appear, and eventually how the beam will behave in the later stages of loading. A numerical procedure for the work presented was adopted in this paper in

which a ‘register-eliminate’ algorithm was developed to predict the cracking process of non-prismatic concrete beams based on the proposed combined-interaction method.

Register-eliminate algorithm

In this procedure, a generation of possible cracks were found in each step of loading based on both interaction approaches (smeared cracks from full-interaction analysis and discrete cracks from partial-interaction analysis). The algorithm was designed to iteratively eliminate less possible cracks (according to the PI and FI analysis described below) in the calculations and register actual cracks among these possible cracks based on the highest possibility. The term “actual crack” is used here for the resulting cracks which are predicted to occur.

The procedure was automated in MATLAB. In the first step, the concrete beam is divided into a number of sections. The moment-curvature and load-slip relationship for each section are found based on full- and partial-interaction analysis respectively. In the partial-interaction analysis described earlier, subsequent cracks are assumed to form at a certain distance from an existing crack. These distances are limited to the primary, secondary or tertiary crack spacing. However, in the combined-interaction method, since crack locations are predicted by both interaction models rather than just partial-interaction, crack spacing can vary. Therefore, the $P - \Delta$ relationship is developed for each section based on a range of possible crack spacing, from closest to farthest possible crack spacing. The closest possible crack spacing is found based on the smallest slip distribution length possible once a crack forms. This is because theoretically no crack will form within the close neighbourhood of an existing crack where slip is not zero. The farthest possible crack spacing is taken as the maximum bond-slip development length because if a crack is further than this distance from an existing crack, it will have no effect on the $P - \Delta$ relationship of the existing crack.

In the next step, increments of load are applied to the member gradually and the deflection profile is calculated based on full-interaction bond assumption up until the onset of the initial crack. The vicinity length (the length of slip distribution region on the left and right-hand sides of the crack) of the initial crack is then found. Outside this region, locations of possible cracks are found using full-interaction analysis. This is done by introducing a cracking index which is equal to the ratio of the applied moment M_A to the cracking moment capacity M_{crf} at each section. Any section with a full-interaction cracking index

M_A/M_{crf} larger than unity indicates a possible crack. Possible cracks based on the partial-interaction method are also found (if any) by finding the applied moment M_A at the cracked section and also the moment needed at the crack to cause another crack to form through a partial-interaction mechanism M_{crp} . If the partial-interaction cracking index M_A/M_{crp} is larger than unity then a possible crack location is indicated. Figure 6a through 6d shows the procedure of eliminating possible cracks and registering actual cracks at two consecutive post-cracking loading stages. The iterative register-eliminate algorithm is also shown in Figure 7.

Next, actual crack(s) are selected among the possible cracks. The full- and partial-interaction cracking indexes are compared. The first modelled crack forms at which the cracking index is highest. The slip distribution around this crack is determined using PI analysis. All possible cracks falling within the vicinity (slip region) of the newly formed crack are eliminated. The section with the second highest cracking index among the remaining possible cracks is then selected as another actual crack and again all possible cracks in its neighbourhood (slip distribution region) are eliminated.

This procedure is continued until all possible cracks are either indicated as actual cracks or eliminated from the calculation. In other words, the calculation stops where there is no possibility for other cracks to form at this stage of loading apart from the actual cracks that are already registered. For additional load increments, the same register-eliminate procedure is applied to find subsequent cracks.

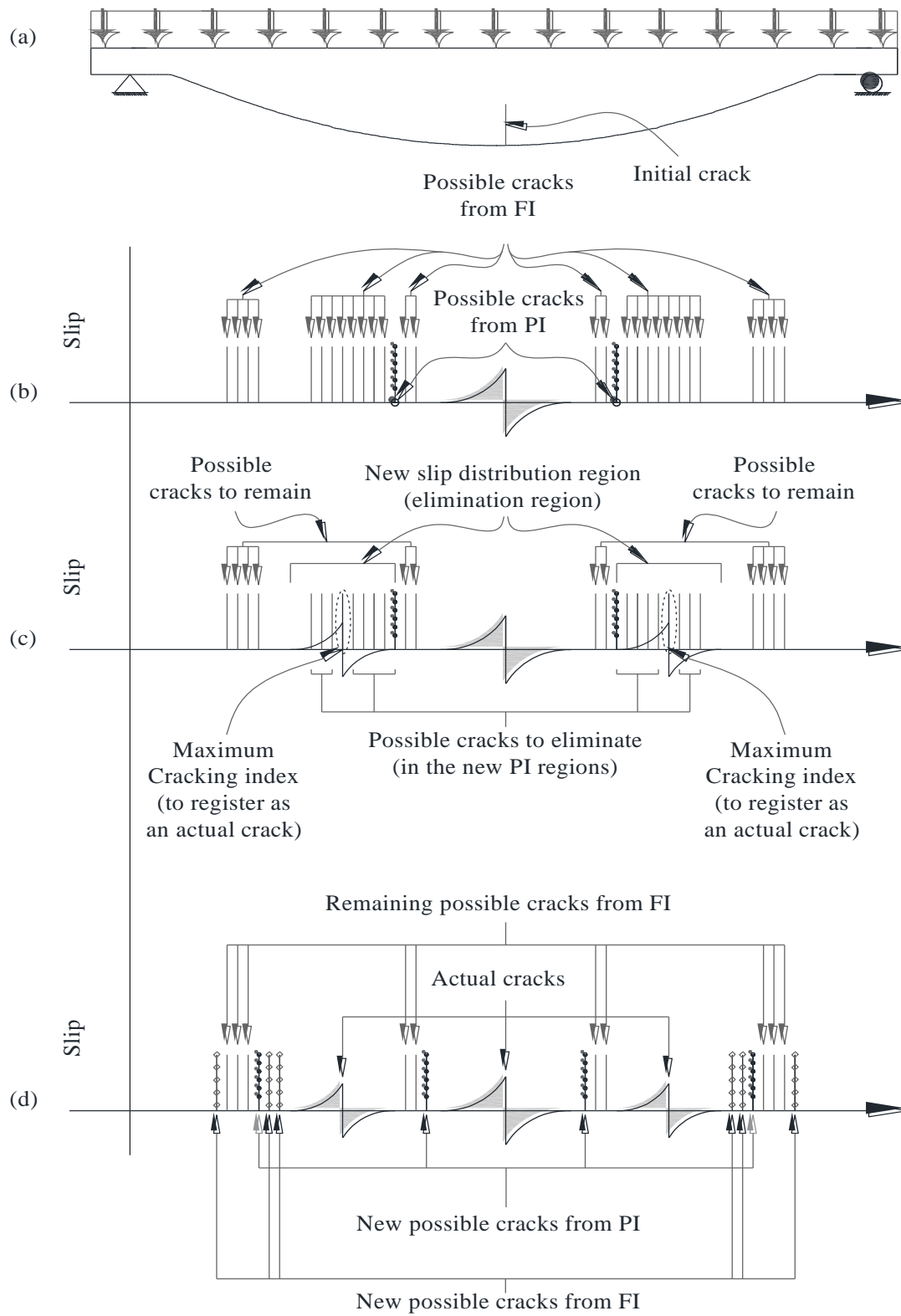


Figure 6: Register-eliminate method: (a) Non-prismatic beam, (b) Finding possible cracks, (c) Elimination of possible cracks and (d) Registering actual cracks.

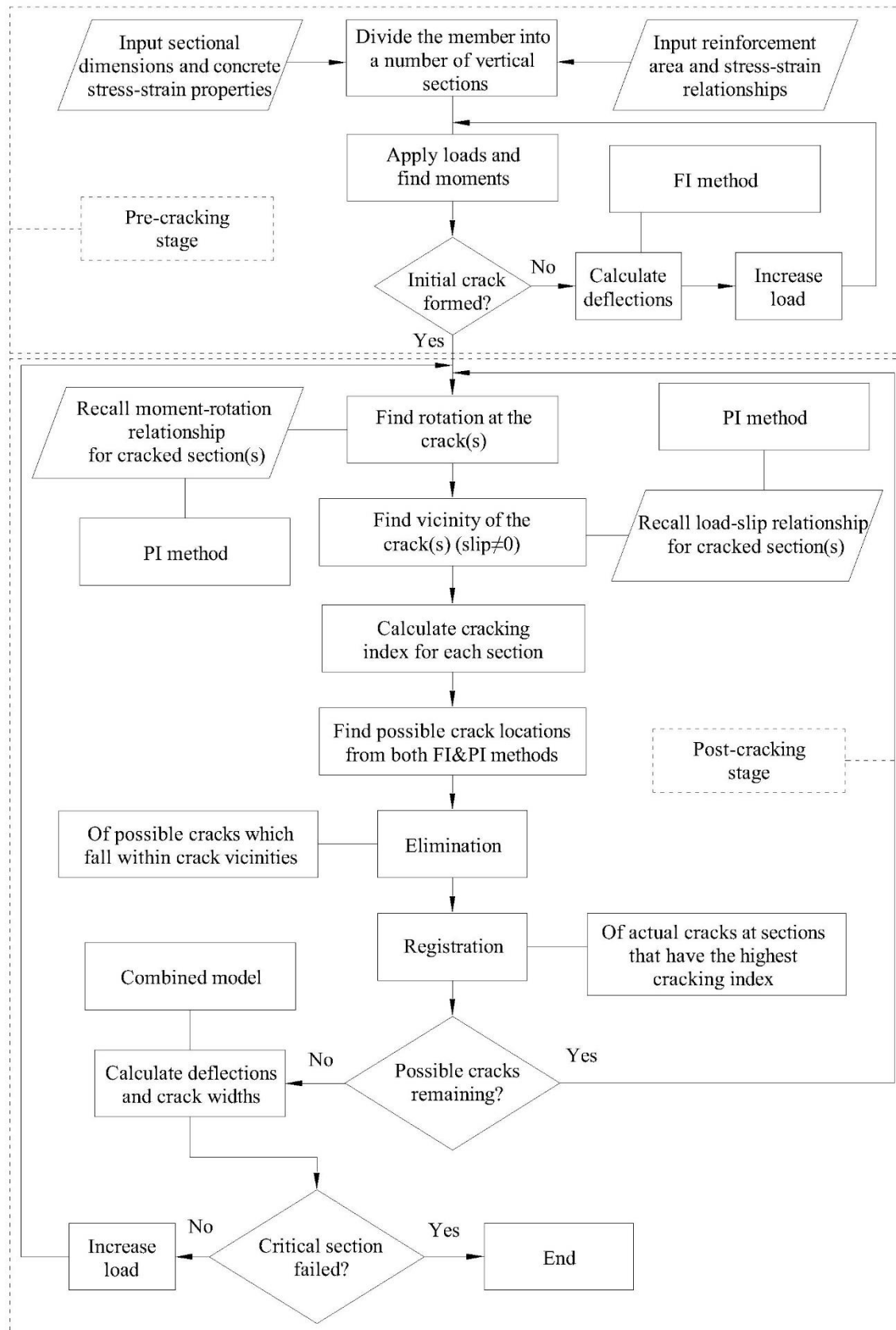


Figure 7: Register-eliminate method

2.3.2 Deflection

The deflection profile is found by combining the contribution to deflections from 1) applying the method of double integration of curvatures to the full-interaction regions where curvature has a non-zero value but slip is zero and 2) applying a discrete crack rotation approach at the actual cracks.

In the early post-cracking stage, deflection from curvatures dominates since there are fewer cracks and full-interaction regions are wider than partial-interaction regions. At later stages of cracking, however, deflections are mainly governed by rotation of discrete cracks rather than by curvature.

Deflection profiles from full-interaction analysis are found by dividing the beam into a number of sections and then recalling moment-curvature relationships for each section, and finding the value of curvature κ corresponding to the applied section moment, M . Once all moment-curvature relationships are known for all sections along the beam, the curvature relating to a specific moment due to loading is integrated once to find the rotations, θ , and these rotations are integrated again to find deflections Δ_f , as described in Equations (3) and (4):

$$\theta = \int_{x=0}^{x=L} \kappa dx \quad (3)$$

$$\Delta_f = \int_{x=0}^{x=L} \theta dx \quad (4)$$

where L is the total length of the beam. A combination of full-interaction and partial-interaction analysis can be used to find deflections as explained below:

$$\Delta = \Delta_f + \Delta_p \quad (5)$$

where Δ is the total deflection value, Δ_f is the contribution to deflections from flexural curvature for uncracked regions which is determined from equations (3) and (4), and Δ_p is the contribution to deflections from discrete crack rotations for the cracked regions and found as below:

$$\Delta_p = \frac{\theta_c L_1 L_2}{L} \quad (6)$$

where L_1 and L_2 are distances from the crack to the two supports, and θ_c is the rotation induced by the crack. When calculating Δ_f , it is important to realise that the beam has curvature values in the full-

interaction regions only, so the value κ should be taken to be zero at those sections that have non-zero values of slip, δ , in the cracked regions as these are accounted for by Δ_p .

3 Verification

In order to verify the models developed in this paper an experimental program was carried out in which two simply supported T-beams were designed and tested under 8-point loading to approximate uniformly-distributed loading. Figure 8 provides details of the designed beam profiles and cross-sections.

The beams were denoted as OS1 and OS2 and had different profiles as they were optimised for different optimisation targets; beam OS1 was optimised for strength only and beam OS2 was optimised for both strength and serviceability. The procedure and results of the optimisation are outside the scope of this paper. Therefore, only the serviceability behaviour of the beams is discussed here.

The beams were cast in timber moulds using concrete of a cylinder design strength of 30 kN. Beam OS1 was tested at the age of 33 days and beam OS2 was tested at 30 days.

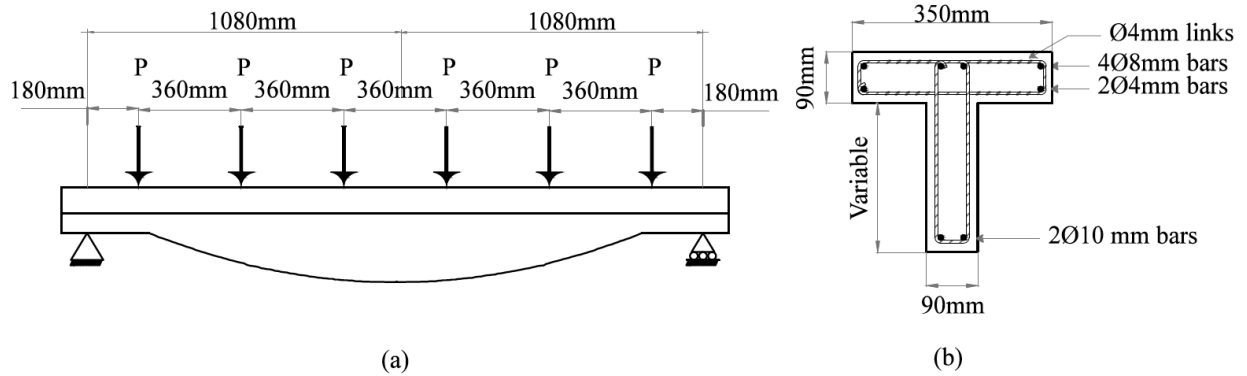


Figure 8: (a) Tested beams profile and test set-up, (b) cross-section of the tested beams

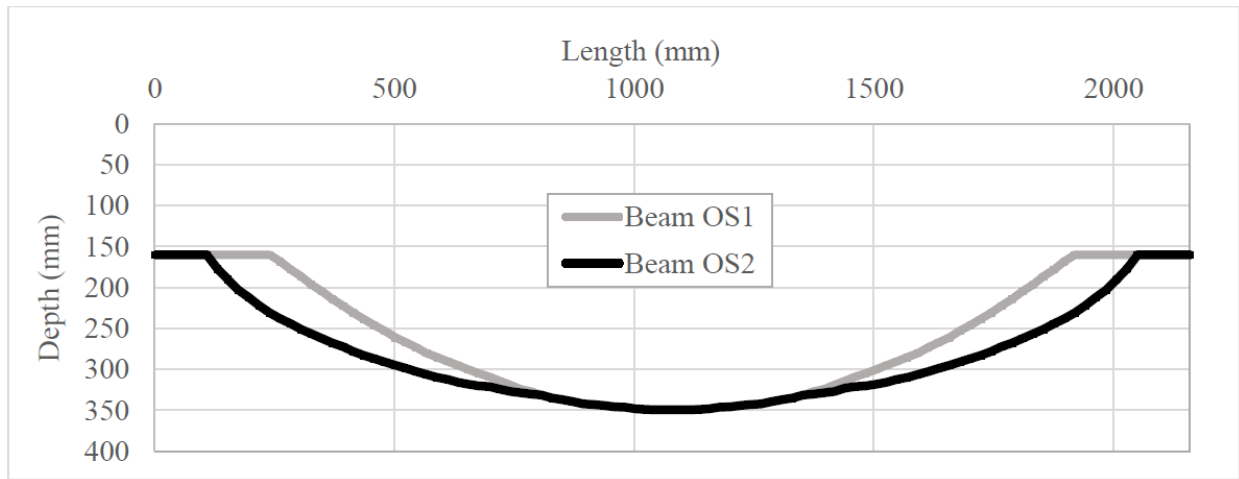


Figure 9: Profiles of the tested beams optimise for ultimate and serviceability limit states

The minimum required depth of the beams near the supports was 160mm to prevent shear failure, and 4mm steel links were placed at 60 mm centres. The as-built longitudinal profiles of the beams are shown in Figure 9. The design limit loads and the predictions of deflections, crack spacing and crack width are shown in Table 1. Since the specimens were subject to laboratory loadings rather than combinations of actual variable and permanent loadings, the service load level is taken to be approximately half of the theoretical ultimate failure load.

Table 1: Predicted ultimate and serviceability limit state values

<i>Beam</i>	<i>Failure mode</i>	<i>Failure load (kN)</i>	<i>Service load (kN)</i>	<i>Maximum deflection at 51 kN (mm)</i>	<i>Average crack width at 51 kN (mm)</i>	<i>Total number of cracks under service loads</i>
OS1	Flexure	105	51	2.6	0.13	16
OS2	Flexure	105	51	2.1	0.10	13

3.1 Material properties

The average yield strength of the longitudinal tension steel bars was 585 MPa and for the compression steel bars it was 510 MPa based on 12 samples for each test. The average cylinder compressive strengths of the concrete for beam OS1 and beam OS2 were 31.7 MPa and 32.6 MPa, respectively, based on three cylinder tests for each beam at the age of 33 and 30 days respectively. The split tensile test was carried out on

100X200mm cylinders; the tensile strength values achieved were converted to axial tensile strength according to (BS EN 1992-1-1, 2004) as follows:

$$f_{ct} = 0.9f_{ct,sp} \quad (7)$$

~~Where~~ where f_{ct} is the axial tensile strength and $f_{ct,sp}$ is the split tensile strength of concrete. The concrete tensile strength was 2.65 MPa for beam OS1 and 2.89 MPa for OS2, also based on three test samples for each beam.

3.2 Material constitutive laws and bond-slip model

Standard models provided by BS EN 1992-1-1 (2004) have been used here to represent the stress-strain relationship of concrete (which characterizes strain-softening) and steel bars under uniaxial compressive and tensile stresses, and the commonly used bond-slip model proposed by CEB-FIP (2010) has been adopted.

3.3 Test method

Loads were applied by hydraulic jacks, as shown in Figure 10. At total load increments of 3kN, deflections were measured under each point load. Crack width readings were taken at every load increment using a high definition crack microscope with a precision equal to 0.02mm. The cracks were measured at the level of tensile reinforcement and at exactly the same points in each loading step.



Figure 10: Test setup for beam OS

3.4 Results

In order to check the validity of the predictions of the combined-interaction method, the data collected from the tests were compared with the analytical data.

The analytical predictions for average and maximum crack widths are compared with the test data for beams OS1 and OS2 in Table 2 and Table 3 respectively. The cracking data at three stages of loading is considered to study the cracking process; at the initiation of cracking at 30kN (33kN for beam OS2 as cracking started at later stages than that of beam OS1), propagation of cracks at 39kN and serviceability, load limit of 51kN.

It can be seen from Table 2 that the prediction of the average crack width values and number of cracks is reasonably accurate at a load level of 33kN and 39kN. The occurrence of all cracks at this load level was predicted by the full-interaction mechanism, while the crack width values were calculated from the partial-interaction analysis, indicating the importance and effectiveness of using a combined approach.

Table 2: experimental and predicted crack width and cracks number values for beam OS1

Load	Average crack width (mm)		Maximum crack width (mm)		Number of cracks		Total crack width (mm)		Discrepancy (Total crack width)	Discrepancy (Maximum crack width)
	Exp.	Pred.	Exp.	Pred.	Exp.	Pred.	Exp.	Pred.		
33	0.07	0.08	0.09	0.12	8	12	0.56	0.96	71%	33%
39	0.09	0.10	0.16	0.13	12	14	1.08	1.4	30%	19%
51	0.10	0.13	0.18	0.15	19	16	1.9	2.08	9%	17%

Table 3: predicted and experimental cracking data of beam OS2

Load	Average crack width (mm)		Maximum crack width (mm)		Number of cracks		Total crack width (mm)		Discrepancy (Total crack width)	Discrepancy (Maximum crack width)
	Exp.	Pred.	Exp.	Pred.	Exp.	Pred.	Exp.	Pred.		
33	0.06	0.07	0.08	0.1	6	9	0.36	0.63	75%	25%
39	0.08	0.1	0.12	0.13	10	11	0.8	1.1	38%	8%
51	0.09	0.1	0.16	0.16	15	17	1.35	1.7	26%	0%

Despite a slight underestimation in the number of cracks in most cases, the experimental average crack widths and their equivalent predictions are in reasonable agreement.

Figures 11 and 12 show the experimental load-deflection relationships compared with the analytical results by the full-interaction and combined-interaction methods for both beams. Here, the load-deflection relationship from the partial-interaction method was not considered for two main reasons: 1) the partial-interaction method is unable to predict load-deflection behaviour of the beams in the pre-cracking stage, and 2) for the non-prismatic beams considered here, a high number of cracks are predicted by the full-interaction mechanism and ignored by the partial-interaction method so that considering the deflection values without accounting for these cracks would not provide a rigorous comparison.

There is generally good agreement shown between both methods. The combined-interaction method predicts service-level stiffness and, hence, deflections more accurately than the full-interaction model. There is no discernible difference between the two methods at higher loads.

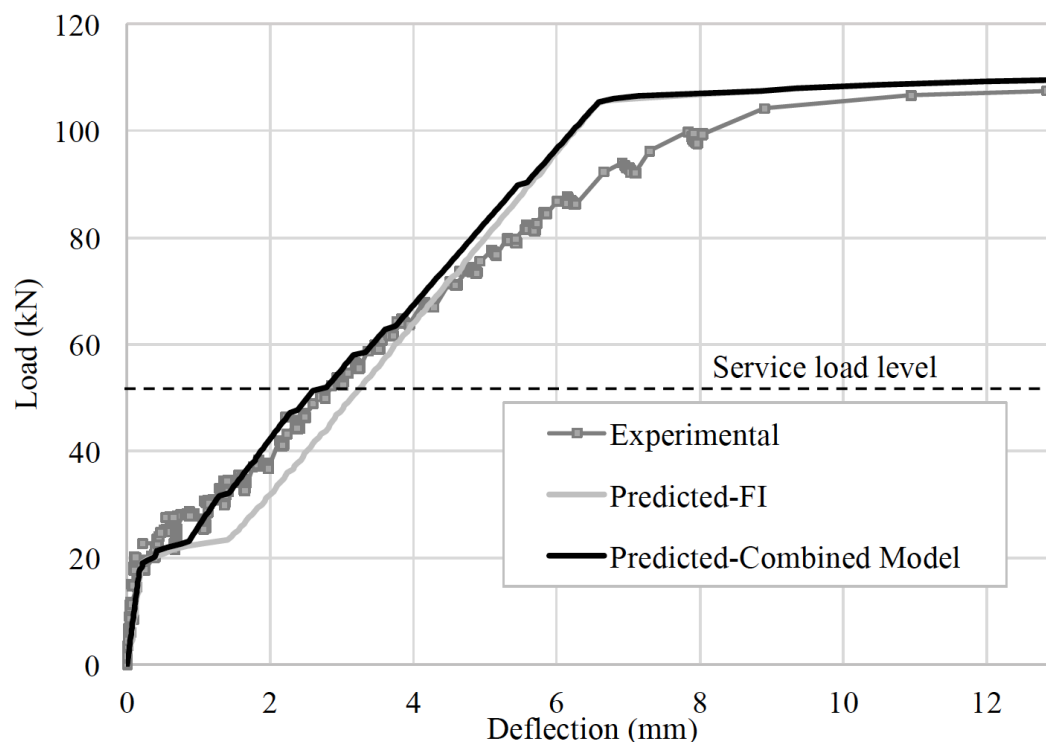


Figure 11: Experimental and predicted load deflection curves for beam OS1

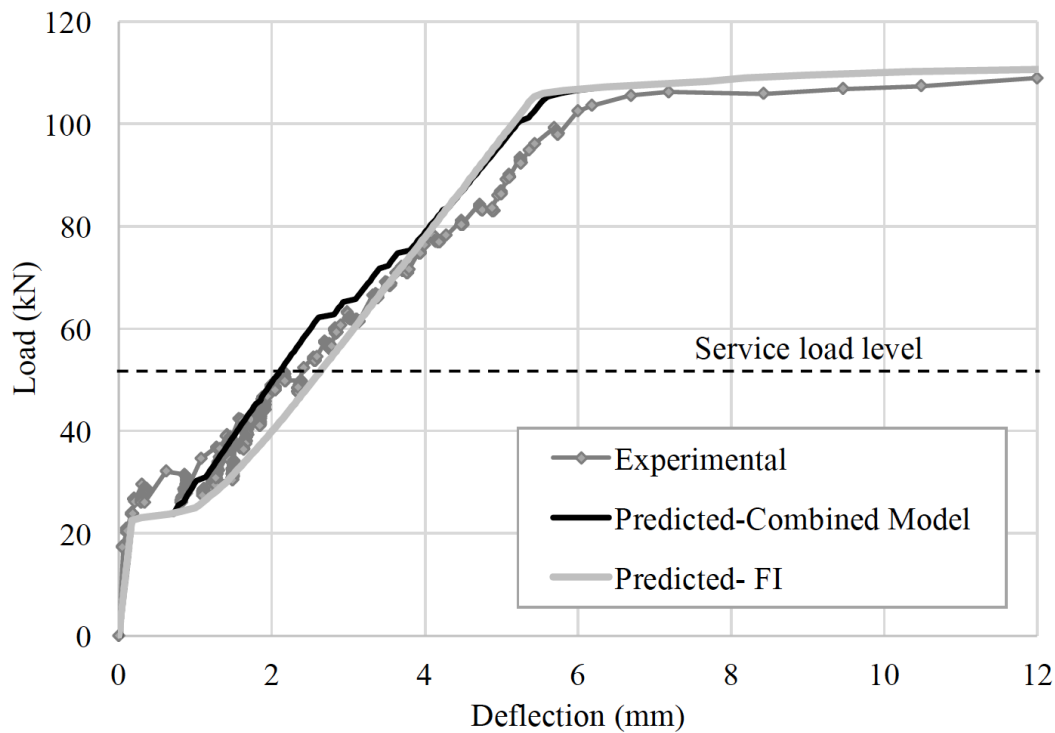


Figure 12: Experimental and predicted load deflection curves for beam OS2

4 Conclusions

Fabric-formed concrete structures offer the opportunity for a new form of architecture which places concrete only where it is needed, thereby saving on concrete quantities. But while full strength envelopes are retained through this approach, overall stiffness of the structure is reduced, so that it is imperative that an accurate predictor of serviceability criteria is developed in order to verify that fabric-formed concrete structures will perform adequately. Therefore, numerical models have been developed in this work in order to predict the behaviour of non-prismatic concrete beams. The prediction model combines full-interaction analysis (where bond slip is not allowed between the reinforcement and the concrete) and partial-interaction analysis (where bond-slip may occur).

The main conclusions of the work presented in this paper are:

- 1- The combined-interaction method developed in this work provides good predictions for deflections at serviceability as it takes into account bond-slip effects.
- 2- In general, predictions for the average and maximum crack widths and the number of cracks are reasonably accurate when compared with experimental results. At serviceability and for beams

OS1 and OS2, the combined-interaction method was ~~about~~approximately 11% and 21% more accurate than the full-interaction method, respectively.

This means that for the first time the relevant serviceability criteria for fabric-formed concrete structures of varying geometry may be predicted adequately. This paper deals with simply supported beams only, however, the combined-interaction method can also be used for continuous beams since since different cracking regions along the beam in positive and negative moment areas can be dealt with separately in this new model.

Future studies on using the combined interaction method for beams reinforced with FRP bars or other types of reinforcement with different bond-slip properties is suggested as different levels of crack propagation at serviceability can be studied using the proposed method. This paper deals with short-term serviceability behaviour of non-prismatic beams without taking into account creep and shrinkage effects, therefore, ~~new~~ further research work considering the effect of these parameters with the proposed combined-interaction model for non-prismatic beams is suggested.

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